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The Halo Density Profiles with Non-Standard N-body simulations

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ABSTRACT

We propose a new numerical procedure to simulate a single dark halo of any size and mass in a hierarchical framework coupling the extended Press-Schechter formalism (EPSF) to N-body simulations. The procedure consists of assigning cosmological initial conditions to the particles of a single halo with a EPSF technique and following only the dynamical evolution using a serial N-body code. The computational box is fixed with a side of $0.5h^{-1}$ Mpc. This allows to simulate galaxy cluster halos using appropriate scaling relations, to ensure savings in computing time and code speed. The code can describe the properties of halos composed of collisionless or collisional dark matter. For collisionless Cold Dark Matter (CDM) particles the NFW profile is reproduced for galactic halos as well as galaxy cluster halos. Using this numerical technique we study some characteristics of halos assumed to be isolated or placed in a cosmological context in presence of weak self-interacting dark matter: the soft core formation and the core collapse. The self-interacting dark matter cross section per unit mass is assumed to be inversely proportional to the particle collision velocity: $\sigma/m_x \propto 1/v$.

Key words: cosmology – dark matter-galaxies: haloes.

1 INTRODUCTION

A detailed understanding of structure formation is one of the central goals of contemporary astrophysics and cosmology. In the popular hierarchical clustering framework (White & Reese 1978) luminous galaxies form by gas cooling and condensing within dark matter halos. These halos merge to build larger structures, hierarchically. In the CDM scenario systematic studies of halo density profiles for a wide range of halo masses were derived by Navarro, Frenk & White (1997; hereafter NFW) who argued that the analytical profile of the form: $\rho(r) = \rho_s(r/r_s)^{-1}(1 + r/r_s)^{-2}$ provides a good description of halo profiles for all halo masses, where r_s is the scale radius which corresponds to the scale at which the slope of the profile is -2.

While predictions of the CDM models have successfully accounted for many observations at large scale scales, increasing interest in testing the predictions at subgalactic and galactic scales has grown in the last few years. Interest began from indications that observed HI rotation curves in the central regions of dark matter dominated dwarf and Low Surface Brightness (LSB) galaxies are at odds with predictions from hierarchical models (Flores & Primack 1994; Moore 1994; Burkert 1995). The models predict circular velocities that increase too rapidly with growing radius compared to the observed profiles of rotation curves. This fact implies a failure of the shape of the predicted halo density profile. Recent high-resolution N-body simulations have shown that, as the numerical resolution is increased, the inner profiles is $\rho \propto r^{-1.5}$, even steeper than the NFW profile (Moore et al. 1999; Fukushige & Makino 2001), exacerbating even more the discrepancy with the observations. On the other hand, Swaters, Madore & Trewella (2000), providing Hα rotation curves for some LSB galaxies, challenged the existence of soft cores for these galaxies. They pointed out that the HI rotation curves are affected by poor spatial resolution, causing a smoothing of the curves in the inner regions (usually named beam smearing). For a sample of late-type dwarf galaxies, van den Bosch & Swaters (2001) show that beam smearing does not allow for distinguishing between the existence or lack of soft cores in late-type dwarf galaxies. However, high-resolution Hα rotation curves of LSB galaxies of de Blok, McGaugh & Rubin (2001) and Marchesini et al. (2002) are in favour of core-dominated halos. Similar results are derived by Salucci & Burkert (2000).

While the question of the existence or lack of shallow cores in galaxies continues to be debated, some authors have revealed the presence of soft cores at the centre of some clusters of galaxies from strong lensing observations in Cl0024+1654 (Tyson, Kochansky & Dell'Antonio 1998) and from X-ray data in Abell 1795 (Ettori & Fabian 2002). Intriguely, clusters showing a soft core do not have a prominent central cD galaxy. In particular Abell 1795 has a central cD galaxy moving far from the centre (Ettori & Fabian 2002). On the other hand, Chandra X-ray data show the lack of any core at the centre of HydraA (David et al. 2001) and a very small core in EMSS 1358+6245 (Arabadjis et al. 2002); both clusters have a central cD galaxy. In relaxed clusters, a dominant central galaxy could play a role making steeper the primordial halo density profile. Accepting the presence of soft cores advocated in Abell 1795 and Cl0024+1654, Firmani and coworkers (2001) have correlated the halo scales from galaxies to galaxy clusters, integrating the available information. Surprisingly, they find that, in this case, the central density of dark halos is independent of the halo mass with a value of $0.05h^2\ M_\odot\ \mathrm{pc}^{-3}$, and that the core radius increases proportionally with the halo maximum rotation velocity.

The presence or the lack of soft cores at the centre of galaxy clusters has interesting consequences for the nature of the dark matter. The lack of any core in clusters makes attractive several scenarios in the debate over the nature of the dark matter. In warm dark matter models, the maximum phase space density of the particles defined as $f_{\max} \equiv \rho_0/v^3$, with ρ_0 the halo central density and v the halo velocity dispersion, has a finite value and is preserved due to the Liouville theorem, implying an increase of the halo central density when the halo mass increases: $\rho_0 \propto v^3$ (Sellwood 2000; Hogan & Dalcanton 2000). In this scenario very massive halos of galaxy clusters are predicted to have high central densities.

A different nature for the dark matter was proposed by Spergel & Steinhardt (2000) in order to overcome the soft core question: if the dark matter is self-interacting, heat transfer towards the central regions triggers a thermalization process in the dark halos avoiding the formation of a cuspy profile. The effects of weak self-interacting dark matter were investigated using N-body simulations on isolated halos (Burkert 2000; Kochaneck & White 2000) and on CDM halos (Yoshida et al. 2000; Davé et al. 2001). These simulations were performed assuming a cross section independent on the relative particle velocity. Ostriker (2000) and Hennawi and Ostriker (2002) have shown a possible inconsistency of the collisional scenario: indeed the model would cause an exorbitant grow on supermassive black holes that imposes a very strict upper limit on the collision cross section. This conclusion derives from their assumption that when black holes seeds have been formed (at $z < 20$) the innermost dark halo shown a NFW density profile. This is not necessarily true, at $z < 20$ collisions have lowered the central density to values at which the growth of the black holes seeds becomes negligible. The other upper limits pointed out by the same authors and concerning the gravothermal catastrophe and the galactic halo evaporation in clusters are consistent with the cross section used in this work. On the other hand, several authors have pointed out some limitations of the collisional dark matter models. Assuming a dark matter cross section independent of the particle relative velocity, Miralda-Escudè (2002) shown that this scenario predicts cluster cores which are too large and round to be consistent with gravitational lensing data.

This work is focussed on the soft core question and explores the possibility that the cross section for the dark matter interaction decreases with velocity $\sigma/m_x \propto 1/v$ as suggested by Firmani et al. 2001 and Wyithe et al. 2001. The focus here is an exploration of halo core properties using N-body techniques in a very weak self-interacting regime. For haloes within a hierarchical context, the cross section value we propose rules out ranges in which the evaporation problem and the core collapse could be significant. We show that if further observational data can confirm evidence of shallow cores in galaxy cluster halos, then self-interacting dark matter with a cross section inversely proportional to the halo dispersion velocity is capable of predicting the existence of soft cores in dwarf galaxies as well as in galaxy clusters. This paper is organized as follows: in section 2 the numerical technique is described, in section 3 for collisionless dark matter particles, we compare the cosmological halo density profiles obtained with our technique to the NFW models for different size halos. Section 4 introduces the Monte Carlo method for weakly interacting particle systems implemented in the code. In sections 5 and 6, the halo density profiles are investigated in a self-interacting dark matter scenario for two different cases: isolated halos and cosmological halos.

2 METHODOLOGY

In all numerical simulations, approximations and compromises are necessary if the calculation is to be completed within a reasonable amount of time. An important choice to make is the effective resolution of the simulation: increasing the spatial resolution requires increasing both the number of particles to prevent two-body effects and the number of timesteps to follow the evolution of smaller structures. For a given simulated volume of universe, the number of particles determines the mass resolution. In cosmological simulations involving only collisionless dark matter the choice of particle number is often driven by computer memory limitations. At present large cosmological N-body simulations have reached the stage where detailed structural properties of many dark matter halos can be resolved simultaneously making use of codes running in parallel and of a very large and expensive number of particles. We use for our simulations the adaptive, particle-particle, particle-mesh $\mathrm{P}^3\mathrm{M}$–SPH code, which is basically the serial publicly released version of HYDRA described in detail by Couchman, Thomas & Pearce (1995). The publicly available serial version of the code does not allow for cosmological simulations of large volumes of universe and the resolution of smaller structures. With the aim of overcoming this difficulty without using supercomputers, we have introduced relevant modifications to the code in order to satisfy the following requirements: *i)* the code has to be reasonably fast, serial, and running on a single processor workstation; *ii)* for each run the code must follow the dynamical evolution of a single isolated or cosmological halo in a limited volume; *iii)* the code has to be capable of describing the properties of collisional or collisionless dark matter halos of different sizes and masses from galactic to galaxy cluster scales with an arbitrary number of particles.

The main change to the code concerns the cosmological initial conditions. Generally cosmological simulations with N-body grid codes begin by fixing a power spectrum of fluctuations, in accordance with a cosmological model on a uniform grid which covers the whole computational box. The fluctuation field is perturbed in Fourier space and the Zel'dovich displacement is assigned to the particles. The grid defines the lowest level of resolution of the simulation. Subsequently, simulations are run with a fixed number of particles and halos are identified in the simulation. Once marked, the selected halos are re-simulated with higher mass and force resolution.

Here, we propose simulations with a different approach. Assuming a spherical symmetry, the dark matter initial distribution at z=100 of a single halo is very close to a uniform density distribution in expansion. A small negative radial density gradient causes each shell to reach the maximum expansion at the dynamical time corresponding to its inner average density. Because of the negative density gradient, the dynamical time is higher in the most external shells. This fact illustrates why the initial negative density gradient is equivalent to a mass aggregation history (MAH) for the halo. In our case, for a given halo present mass, a MAH is obtained from the EPSF developed by Avila-Reese, Firmani and Hernández (1998; hereafter AFH). Using the dynamical time of each shell implicit in the MAH we recover the initial density gradient, i.e. the initial condition at z=100. The stochastic nature of the density fluctuations induces a stochastic structure on the MAH. We have limited our analysis on the average MAH from which a given present halo mass has grown. This initial condition for the description of the cosmological evolution of growing halos offers clear advantages with respect to the simulations of isolated halos starting with a NFW or a Hearnquist density profile. However, because the mass accretion is described in a spherical symmetric frame, in this scheme the tidal destructive action of strong mergers on a central dense halo core is not taken into account. This fact establishes a limit on our approach that works in the sense to make more difficult the formation of inner shallow density profiles. Because of our goal is to explore the capability of self-interaction to create shallow cores, our results should be strengthened if a detailed cosmological simulation is made. Our result will be particularly correct in the case of dwarf and LSB galaxies which, being fragile disks and field objects, their MAH could not be affected by major mergers. In the case of galaxy clusters our result is purely indicative and obeys to a completeness criterion. In fact in this case major mergers may amplify the formation of shallow cores as well as they may introduce significant deviations from the spherical symmetry. Taking into account the previous discussion and considering that our goal is to analyze the ability of self-interaction to create shallow cores, we concentrate our attention on the average MAH. The lack of any stochastical information from observations makes unnecessary any analysis which takes into account the stochastical nature of MAHs.

The following steps are taken:

- We assign to the halo an initial density profile described by a collection of concentric shells. For each shell, the MAH of the halo gives: the time of maximum expansion t_{max}, the radius of the maximum expansion, the initial radius at a given redshift, and the cumulative mass within the shell.
- The dynamical evolution of the halo within a non-linear regime is followed by the N-body code.

At the beginning of our N-body simulations the radius of each particle is interpolated to the initial radius, determined by the halo MAH at a cumulative value of the mass. We have used different techniques to distribute the particles (the halo mass) within the shells at the beginning of the simulation, with a grid or with isotropic random direction. In the first case, particles are assigned on a spherical mesh points and are originally equally spaced on the mesh, then the distance of each particle from the centre is computed according to the halo MAH. In the second case, for each particle the radius is isotropically assigned using a Monte Carlo routine, while the magnitude of the radius is fixed by the cosmological MAH. Both methods in assigning the particles at the initial conditions have limits and advantages. The first method uses a grid to assign particles and thus it is closer to the standard assignment used in the original version of the code. We have used it for the cosmological collisionless and collisional runs. However, in some cases, a weak filament is seen in the virialized halo. This elongated structure is an artificial numerical effect, and some tests have shown that it is present already in the original version of the code. On the other hand, the second technique prevents the formation of elongated structures but since the particles are assigned with random isotropic directions, they rapidly clump, due to the gravity, to form substructures at very early times that survive during the virialization of the halo. These substructures are spurious. Thus, for the cosmological simulations we prefer to assign particles to a grid. The Hubble flow at $z = 100$ was assigned to the particles, as initial velocity. The initial value of z has been chosen with the criterion to avoid any spurious formation of soft cores.

3 MODELLING COLLISIONLESS CDM HALOS

The focus of the approach proposed here is on the exploration of density profiles of halos of any size and mass. We show now that by making use of the N-body code with the initial conditions assigned with our new numerical technique, we can reproduce the NFW density profile for a single collisionless dark matter cosmological halo of any mass, after the code is run for a few hours (CPU time) on a standard workstation.

We carried out simulations of galactic halos as well as galaxy cluster halos within a flat ΛCDM universe with matter density $\Omega_m = 0.3$, cosmological constant $\Omega_\Lambda = 0.7$, and rms linear fluctuation amplitude in $8h^{-1}$ Mpc spheres of $\sigma_8 = 1$. Each simulation followed the trajectories of $N = 80,000$ dark matter particles within a physical cube of side 0.5 h^{-1} Mpc from z=100 to the present. The Hubble constant is assumed to be of 65 km s^{-1} Mpc^{-1} (h=0.65). Tests with more particles did not change the results. One refinement level was introduced in the region of highest mass density.

On galactic scales, for a halo mass of $10^{11}h^{-1}$ M$_\odot$ the mass per particle used is $m_p = M/80,000 = 1.25 \cdot 10^6 h^{-1}$ M$_\odot$ and the softening length is fixed in order to obtain a spatial effective resolution of ≈ 1 h^{-1} kpc. On scales of

galaxy clusters we have carried out simulations of a single halo with $M = 10^{15}h^{-1}\ M_{\odot}$ with a mass resolution of $m_p = 1.25 \cdot 10^{10}h^{-1}\ M_{\odot}$ and an effective spatial resolution of $\approx 20\ h^{-1}$ kpc.

We stress that the cluster simulation was obtained in the same cube of side 0.5 h^{-1} Mpc. In fact, as gravitation is a scale invariant physical process, a halo of any mass and size may be simulated in the same fixed volume. Of course, because of the hierarchical process of mass accretion, the mass aggregation history of a galactic halo is different from the galaxy cluster halo merging history. Thus, once the MAH is generated for a halo of galaxy cluster size, we can evolve the dark halo by N-body simulations by rescaling the mass M and the radius R to any mass M_0 and initial radius R_0 with the following relations:

$$M = mM_0, \tag{1}$$

$$R = rR_0, \tag{2}$$

$$t = t_0(r^3/m)^{0.5}, \tag{3}$$

$$v^2 = v_0^2\,(m/r)\,, \tag{4}$$

where m, r are scaling parameters. This work simulates galaxy cluster halos by rescaling the cluster mass to mass M_0 of a galactic halo with $10^{11}\ h^{-1}M_{\odot}$.

Simulated density profiles of collisionless dark matter are shown in the top panel of Figure 1 for two halos with the following masses: $M = 10^{11}\ h^{-1}M_{\odot}$ (filled circles) and $M = 10^{15}\ h^{-1}M_{\odot}$ (open circles). Solid lines are the NFW profiles for the same halos. The good agreement between the halo density profiles obtained with our method and the NFW model is encouraging. The velocity dispersion radial profiles corresponding to the same halos are drawn in the bottom panel of Figure 1 with filled symbols corresponding to the galactic halo of $M = 10^{11}\ h^{-1}M_{\odot}$ and open symbols to galaxy cluster halo of $M = 10^{15}\ h^{-1}M_{\odot}$, as derived by our simulations. The overlapped solid lines are the radial velocity dispersion for the NFW model with the same mass.

A more quantitative comparison to the NFW model for the two halos is obtained deriving values for the concentration parameter, defined as the ratio between the virial radius and the scale radius $c = r_{vir}/r_s$ (Navarro, Frenk & White 1997). For the halos of $M = 10^{11}\ h^{-1}M_{\odot}$ and $M = 10^{15}\ h^{-1}M_{\odot}$ we estimate c=9.5 and 5, respectively for our realizations. The concentration values of our run are plotted in the Figure 2 as filled points and compared to the mass-concentration relation of the NFW model (dashed line) [⋆]. The halos are a little more concentrated than the average concentration predicted by the NFW model. However, high-resolution N-body simulations have shown a spread around the average mass-concentration relation (Wechsler et al. 2002), thus the concetrations we find in our two realizations are inside the spread.

⋆ The mass-concentration relation of the NFW model is derived using the program kindly made available to the community by J.Navarro.

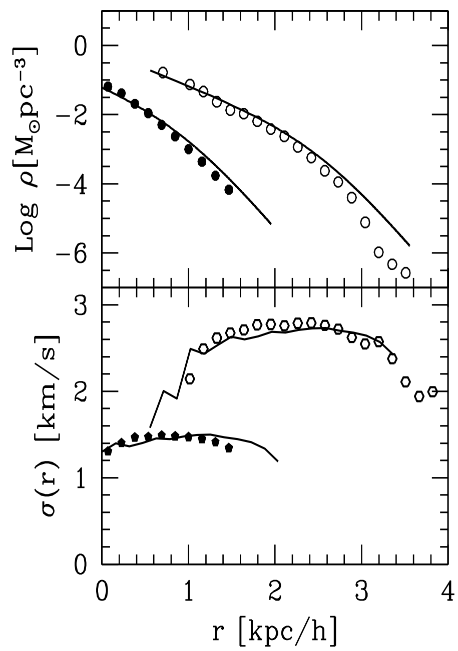

Figure 1. In the top panel simulated density profiles of collisionless dark matter are shown using non-standard N-body simulations for halos with masses: M= $10^{11}h^{-1}M_{\odot}$ (filled circles) and M= $10^{15}h^{-1}M_{\odot}$ (open circles). A Hubble constant of 65 km s^{-1} Mpc^{-1} is adopted. Solid lines are the NFW profiles for the same halos. Corresponding velocity dispersion radial profiles are drawn in the bottom panel with filled symbols corresponding to the galactic halo and open symbols to the galaxy cluster halo. The solid lines are the radial velocity dispersion for the NFW model.

4 MODELLING COLLISIONAL CDM HALOS

The idea of a collisional dark matter suggested by Spergel & Steinhardt (2000), is capable of producing soft cores and, at the same time, preserves the success of CDM models in explaining the observed properties of the universe at large scales. In fact, in the hierarchical framework of structure formation, the halo continuously accretes mass by merging. However, collisions between particles cause heat transfer inwards. Since its negative heat capacity, the core gains energy and expands, producing a soft core. However, the idea of weak self-interacting dark matter poses the following question. What is the final equilibrium configuration of the halo? The initial state is a halo described by a NFW model far from thermal equilibrium, and the final state is a halo with a thermalized core.

We explore the dynamical properties of collisional dark matter studying isolated or cosmological halos of any mass using N-body techniques. We use a Monte Carlo technique to implement the self-interaction of the dark matter particles in collisionless N-body code. The scattering process between the dark particles is implemented with the following algorithm. Within a timestep Δt, the probability P_i of

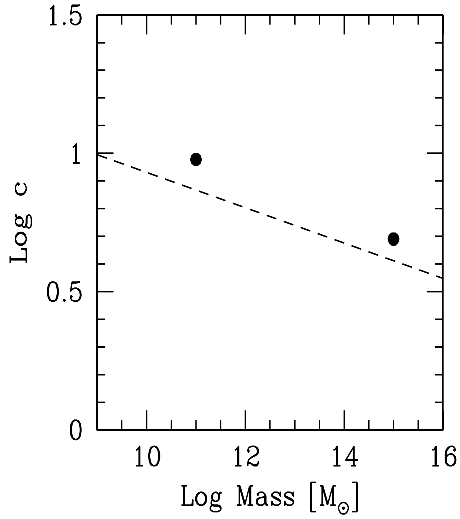

Figure 2. Concentration parameters c versus the halo mass for the two halos of M= $10^{11}h^{-1}M_\odot$ and M= $10^{15}h^{-1}M_\odot$ as obtained from our runs (filled points). The dashed line is the halo Mass-concentration relation of the NFW model.

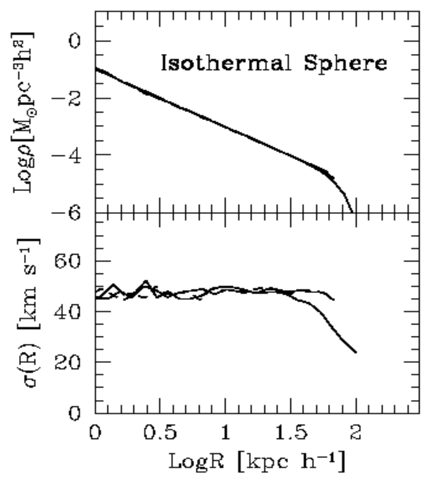

Figure 3. In the top panel the time evolution of the density profile is shown for a halo of mass M= $10^{11}h^{-1}M_\odot$ described by a singular isothermal sphere. This run uses collisionless dark matter particles. After several timescales, the density distribution is represented by the solid line, while the short dashed line is the profile at the beginning of the simulation (overlapped). In the bottom panel the corresponding radial velocity dispersion is shown.

a particle with velocity $\vec{v_i}$ to interact with another particle is:

$$P_i = \Gamma_i \ \Delta t, \tag{5}$$

with Γ_i the scattering rate for a particle with velocity v_i:

$$\Gamma_i = \Sigma_j^N \left(\frac{\sigma}{m_x}\right) \frac{m_j[\vec{v_j} - \vec{v_i}]}{V}, \tag{6}$$

where $\Sigma_j^N m_j/V$ is the estimate of the local density, with $V = 4\pi r^3/3$. Although a constant cross section is used by several authors, here we assume a cross section inversely proportional to the particle velocity dispersion as found to be justified by the observations in the central densities of galaxies and galaxy clusters (Firmani et al. 2001; Wyithe et al. 2001)

$$\left(\frac{\sigma}{m_x}\right) \cdot v \approx \text{const} \tag{7}$$

Within a timestep the algorithm is as follows:

i) For each particle the interaction probability P_i is computed.
ii) For each scattering event the collision is with one of the nearest particles (N=16, but increasing N does not change the result).
iii) Once a particle is selected for the interaction, the scattering is isotropic and the diffusion is computed.

Concerning the diffusion process, for particles with centre-of-mass velocity $\vec{v_B} = (\vec{v_i} + \vec{v_j})/2$ and velocity difference $\vec{v_0} = (\vec{v_i} - \vec{v_j})/2$, a random direction $\vec{e}$ is selected and new velocities are assigned to the colliding particles i and j

$$\vec{v_i} = \vec{v_B} + \frac{\vec{e}}{|e|}|\vec{v_0}|, \tag{8}$$

$$\vec{v_j} = \vec{v_B} - \frac{\vec{e}}{|e|}|\vec{v_0}|. \tag{9}$$

The energy and linear momentum are preserved. The numerical method is similar to the algorithm implemented by Burkert (2000) and Yoshida et al. (2000) with the difference that the cross section is dependent upon the particle velocity. For the cross section we assume:

$$\left(\frac{\sigma}{m_x}\right) = 10^{-24} \left(\frac{100 \text{ km s}^{-1}}{v}\right) \frac{\text{cm}^2}{\text{GeV}}. \tag{10}$$

In order to simulate a galaxy cluster halo with collisional dark matter, the cross section value was rescaled as well as the radius, velocity, time, and mass according to eqs.(4) to (6) inclusive.

5 ISOLATED HALOS AND CORE COLLAPSE

For isolated halos the balance between continuous mass aggregation due to the halo merging history and the thermalization process induced by collisions fails. In this section we

investigate whether this case can produce core collapse with a consequent gravothermal catastrophe in a short time.

First, we test if the code is able to preserve the state of dynamical equilibrium in a model. We have simulated a halo with *collisionless* dark particles and a mass profile of an isothermal sphere. This choice is related to the known properties of this model. In the top panel of Figure 3, the time evolution of the density profile is shown for several dynamical timescales. The density profile preserves the shape for all times. After several timescales, the density distribution is represented by the solid line, while the short dashed line is the profile at the beginning of the simulation. In the bottom panel the corresponding radial velocity dispersion is shown.

Note that the isothermal sphere was settled by assigning to the particle velocities with amplitudes consistent with the Maxwellian velocity distribution function, $f(v)$ of the model. In fact, in this case the initial profile is at equilibrium and does not show any adjustment during dynamical evolution. The result is remarkable: the dynamical equilibrium of the model is preserved for all times. The only modification to the profile is after many dynamical timescales in the outer parts due to a evaporation process of the particles. Furthermore, this model tests the softening radius fixed in our simulations. For a galactic halo of 10^{11} $h^{-1}M_{\odot}$ the softening radius was fixed in order to obtain a spatial effective resolution of 1 h^{-1} kpc.
We have followed the evolution of dark density profiles for a galactic halo of 10^{11} $h^{-1}M_{\odot}$ formed by collisional dark matter and characterized by the following initial mass distribution: the Hernquist model and the King profile.

5.1 The Hernquist profile

An isolated galactic halo model described by an initial Hernquist profile (Hernquist 1990) has attracted considerable attention in the study of self-interacting dark matter.

For this model characterized by a central density cusp ($1/r$) similar to a NFW profile, Burkert (2000) and Kochanek & White (2000) argue that soft cores are produced in a short dynamical timescale induced by collisions. However the shallow cores so created are unstable and a singular isothermal sphere is produced as a final halo density profile.

We set up the initial conditions as a Hernquist profile with the halo mass of 10^{11} $h^{-1}M_{\odot}$ and a characteristic scale radius $a = 50h^{-1}$ kpc. Following the analytical profile, particle positions are distributed isotropically and the magnitude of the velocity is computed from the energy distribution function, $f(E)$, of this model, also assigned with an isotropic distribution.

First, we run a model without collisions between particles and we have shown that the shape of the profile is not modified during dynamical evolution. The dynamical equilibrium is preserved. The dynamical time is defined:

$$t_{\rm dyn} = 4\pi\sqrt{\left(\frac{a^3}{GM}\right)}; \tag{11}$$

thus, the dynamical time is $t^H_{\rm dyn} = 6.7$ Gyr ($t^H_{\rm dyn}$ is referred to the Hernquist model). In the top-left panel of Figure 4, the time evolution for the Hernquist profile is shown for self-

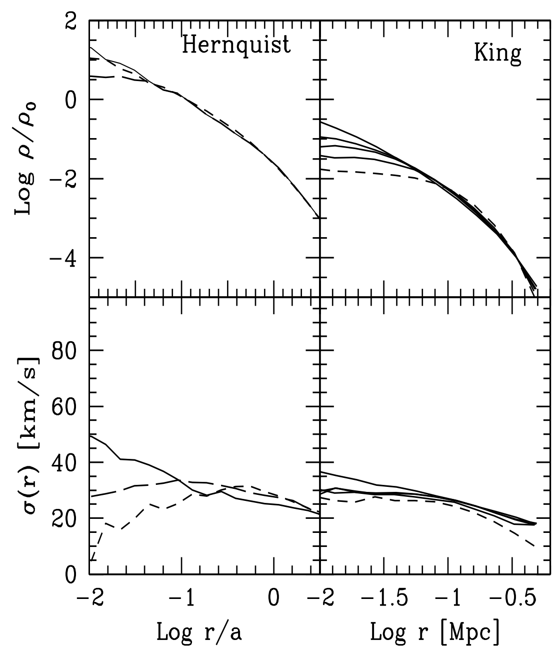

Figure 4. *Left panels.* The time evolution for the Hernquist profile is shown for self-interacting cross section value obtained assuming a cross section inversely proportional to the halo dispersion velocity, $(\sigma/m_x) \cdot v_{100} = 10^{-24}$ cm^2 GeV^{-1}, where v_{100} is the halo dispersion velocity in units of 100 km s^{-1}. The initial Hernquist profile is shown as a short-dashed line. After 2.25 dynamical timescales a minimum central density is reached (long-dashed lines) and the core collapse after 5 dynamical timescales (solid lines). Corresponding velocity dispersion radial profiles are shown in the bottom. *Right panels.* The time evolution of the King profile is plotted. Dashed lines are the initial King profile (t=0). After a while, the initial flat core reduces its size towards core collapse. In the plot the time evolution halo density profile is shown, from the lower solid line to the higher solid line representing the profile after 0.8 $t^K_{\rm dyn}$. In the bottom panel of the same figure the radial velocity dispersion profile is shown for the model.

interacting cross section value obtained assuming a cross section inversely proportional to the halo dispersion velocity, $(\sigma/m_x) \cdot v_{100} = 10^{-24}$ cm^2 GeV^{-1}, where v_{100} is the halo dispersion velocity in units of 100 km s^{-1}. The initial Hernquist profile is shown as a short-dashed line. After 2.25 dynamical timescales a minimum central density value is reached (long-dashed line), increasing after 4 dynamical timescales and reaching a lack of the core after more than 5 dynamical timescales (solid line), due to the collisions between particles. We also run simulations with very high cross section values, finding that once the soft core is formed, it disappears in a short time, in agreement with the work of Kochaneck & White (2000) and Burkert (2000), when the cross section was adopted to be independent of the particle collision velocity. Thus, high cross section values seem to work on a catalyst of the core collapse.

5.2 The King profile

We have simulated an isolated halo of $10^{11}\ h^{-1}M_{\odot}$ with a King model mass distribution. In fact, the dark density profile of LSB and dwarf galaxies is well matched by a King profile with form parameter $P = 8$ (Firmani et al. 2001). We test whether starting with a isothermal configuration and a shallow core the isolated halo undergoes core collapse due to the presence of collisions between the dark particles. We want to follow the halo core evolution through isothermal equilibrium states. After assigning isotropic positions to the particles, we pay attention that at the beginning of the simulation the profile is at equilibrium, and we calculate the velocity magnitude from the King distribution function $f(v)$. Again, particle velocities are isotropically distributed with a Monte Carlo routine.

First, we have run the model with $\sigma/m_x = 0$. The shape of the King profile is preserved at all times. We now run the model in a strong self-interacting regime: $(\sigma/m_x) \cdot v_{100} \simeq 5{\cdot}10^{-22}$ cm^2 GeV^{-1}. In fact, for weak cross sections, we have verified that this model evolves slowly towards core collapse. In this case, a large cross section value is assumed in order to hasten core collapse. The dynamical time depends upon the adopted concentration for the King model. Defining the dynamical time as above, we find that $t^K_{\rm dyn} \approx 4\ t^H_{\rm dyn}$ (where $t^K_{\rm dyn}$ is referred to the King model and $t^H_{\rm dyn}$ to the Hernquist model, respectively). In the top-right panel of Figure 4, the time evolution of the King profile is plotted. Dashed lines are the initial King profile (t=0). After a while, the initial flat core reduces its size towards core collapse. In the plot the time evolution halo density profile is shown, from the lower solid line to the higher solid line representing the profile after $0.8\ t^K_{\rm dyn}$. In the bottom panel of the same figure the radial velocity dispersion profile is shown for the model.

It is interesting to note that any reasonable value for the cross section (weak or strong) produces core collapse. Weaker collisional regimes will produce core collapse in a longer time. Of interest is the evidence that a dark halo described by a King profile evolves via equilibrium states towards the gravothermal catastrophe under the effect of collisions.

6 COLLISIONAL HALOS IN A HIERARCHICAL COSMOLOGICAL FRAMEWORK

The question we investigate in this section is: assuming a modest self-interacting cross section with $(\sigma/m_x) \cdot v_{100} = 10^{-24}$ cm^2 GeV^{-1} are halo soft cores produced by collisional dark matter in *a cosmological framework* stable or can they collapse? In other words, can dark halo density profiles be soft at the centre and changing slowly their slope reaching the NFW shape or steeper, due to the collision, in a Hubble time?

Using the N-body technique described above we run simulations for a galactic halo and a cluster halo. In Figure 5 we show the dark matter density profiles of a halo of M= $10^{11}\ h^{-1}M_{\odot}$ as obtained in a collisional CDM model. The halo have been modelled assuming a energy dependent cross section with the same value as above. In the top panel the short dashed line shows the halo density profile after 7.5 Gyr, while the long-dashed line is the halo mass density distribution after a Hubble time. In the same plot the solid line is the NFW profile.

Note that within a Hubble time a modest self-interaction cross section value is able to create soft cores. No core collapse occurs within a Hubble time.

The bottom panel of Figure 4 shows the corresponding halo radial dispersion velocity profiles. The velocity distribution of the particles reaches roughly a constant velocity dispersion under the effects of collisions resulting in a central non-singular isothermal density profile.

In Figure 6 the dark density profile is shown for a halo of $10^{15}\ h^{-1}M_{\odot}$. The long-dashed line is the halo density profile after one Hubble time and the solid line is the NFW profile. Even in this case the halo does not undergo the core catastrophe within a Hubble time. We interpret this result assuming that this is the case in which any trend towards the gravothermal catastrophe is avoided by the competition between a mass aggregation determined by a *continuous* halo merging history and a thermalization process by collisions. In fact we assign to the particles cosmological initial conditions assuming that the halo merging history is a continuous process.

In Figure 7, the central densities and the core radii

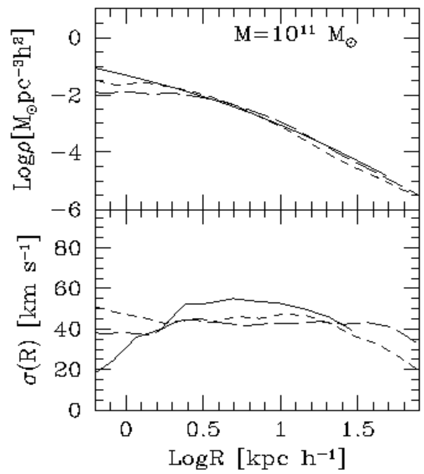

Figure 5. In the top panel the dark matter density profiles of a halo of M= $10^{11}\ h^{-1}M_{\odot}$ are shown as obtained in a collisional CDM scenario. The halo have been modelled assuming a energy dependent cross: $(\sigma/m_x){\cdot}v_{100} = 10^{-24}$ cm^2 GeV^{-1}. Short dashed lines are the halo density profile after 7.5 Gyr, while long-dashed lines are the halo mass density distribution after a Hubble time. In the same plot the solid line is the NFW profile. In the bottom panel the corresponding halo radial dispersion velocity profiles are shown.

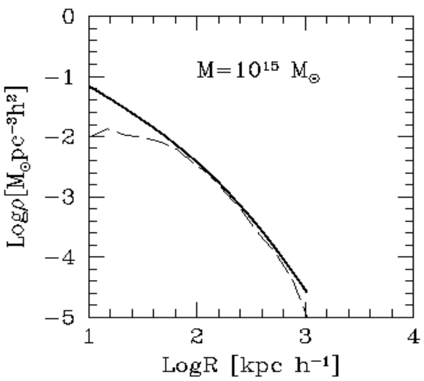

Figure 6. The dark density profile is shown for a halo of 10^{15} $h^{-1}M_{\odot}$, using a energy independent cross section: $(\sigma/m_x)\cdot v_{100}\sim 10^{-24}$ cm^2 GeV^{-1}. The long-dashed line is the halo density profile after one Hubble time and the solid line is the NFW profile.

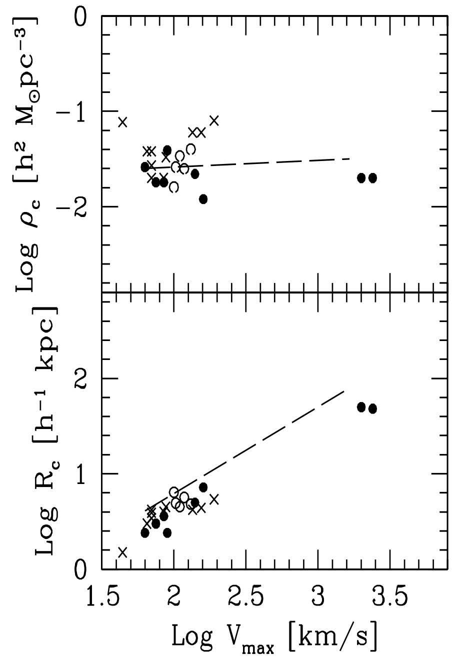

Figure 7. The halo central densities and the core radii predicted by the models are compared with those inferred from HI rotation curves of dwarf and LSB galaxies (diagonal crosses), Hα rotation curves from Swaters et al. (2000), taking into account the halo adiabatic contraction (open circles) and Hα rotation curves of LSB and dwarf galaxies (filled circles) by Marchesini et al. (2002). The filled circles on the right of the panels are two galaxy clusters with evidence of soft cores: Cl 0024+1654 and Abell 1795. Dashed lines are the halo central densities and core radii as a function of the maximum circular velocity predicted by self-interacting dark matter with $(\sigma/m_x)\cdot v_{100}\simeq 10^{-24}$ cm^2 GeV^{-1}.

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A feature of self-interacting dark matter, with any cross section is to produce halos that are more spherical than the CDM halos, due to the isotropic nature of collisions. Thus, collisions tend to isotropize the velocity ellipsoid. Consequently the shapes of self-interacting dark matter halos and CDM halos should be different and provide an interesting observational constraint to discriminate the two scenarios. Miralda-Escudé (2002), providing lensing data for the galaxy cluster MS2137-23, shown that at a radius $R\approx 70$ kpc, anisotropies start in the velocity ellipsoid, constraining the cross section between dark particles to be less than 0.02 cm^{2}g^{-1} in order to create anisotropies in agreement with the observations.

Davé et al. (2001) studied the the axisymmetry and the flattening for galactic halos using N-body simulations with constant cross section between dark particles. After the completition of this work, a study of N-body simulations of self-interacting dark halos appeared by Colín et al. 2002, in which the authors analyse the ellipticities in the halos, even using a cross section inversely proportional to the halo dispersion velocity. Due to the limitations of our method, cosmological triaxiality and a comparison to the lensing data cannot be developed within our approach. Hovewer the effects of the self-interaction on an elongated structure artificially introduced in the early evolution of the halo have been explored. We define the axisymmetry q and the flattening s, following the same prescription used in Davé et al. (2001). A inertia tensor is defined:

$$M_{ij}=\Sigma\frac{x_i x_j}{r^2};\qquad r^2\equiv\left(x_1^2+\frac{x_2^2}{q^2}+\frac{x_3^2}{s^2}\right)\tag{12}$$

with the sum is over all particles with coordinates (x_1,x_2,x_3) and distance r from the halo centre and s and q the axis ratio:

$$q=\left(\frac{M_{yy}}{M_{xx}}\right)^{1/2};\quad s=\left(\frac{M_{zz}}{M_{xx}}\right)^{1/2}\tag{13}$$

with M_{xx},M_{yy},M_{zz} are the eingenvalues of the inertia tensor M.

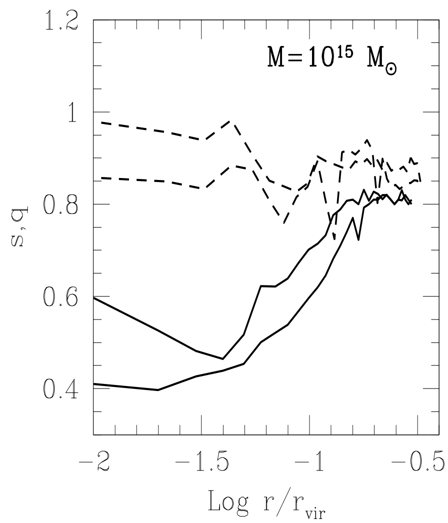

Figure 8. Axisymmetry q and flattening s for a cluster halo of M=10^{15} $h^{-1}M_{\odot}$ versus r/r_{vir}. Solid lines represent s (lower line) and q (higher line) runs with collisionless dark matter halos. Dashed lines are simulations with collisional dark matter with cross section $(\sigma/m_x)\cdot v_{100}\simeq 10^{-24}$ cm^2 GeV^{-1}.

Since the observational constraints on anisotropies existence come from galaxy cluster we feel more interesting to analyse the galaxy cluster scale halo with M=10^{15} $h^{-1}M_{\odot}$ and to assume the same cross section capable to reproduce the soft core in agreement with the observations of Cl0024+1654, $(\sigma/m_x)\cdot v_{100}\simeq 10^{-24}$ cm^2 GeV^{-1}. Figure 8 shows how the galaxy cluster halo is more spherical if a self-interacting scenario is working. For a cluster with $r_{vir}=1.5$ Mpc any early triaxiality desappears inside a region of roughly 120 kpc (for h=0.65). This is not surprising because particle collisions induced by self-interacting dark matter tend to recover the spherical symmetry. However in realistic cases major mergers may be a continuous source for anisotropies, even within a few hundred kpc from the centre.

7 DISCUSSION AND CONCLUSION

We have explored the dark halo density profiles in a collisional dark matter scenario studying isolated or cosmological halos of any size and mass. This study presents two key points: *i)* we use a modified N-body code where the cosmological initial conditions are assigned with a new approach; and *ii)* we use a self-interacting cross section inversely proportional to the particle collision velocity.

Our results may be summarized as follows:

• Concerning the dynamical study of isolated halos we have followed the evolution of different initial profiles: the Hernquist model and the King profile. Surprisingly, the virialized halo density profiles for all models undergo core collapse induced by collisional dark matter. Once the core size is reduced, core collapse is a very rapid process. The shape of the cross section (constant or dependent upon the particle collision velocity) does not affect core collapse. Our N-body simulations performed with a cross section dependent on the particle velocity dispersion produce similar results to the N-body simulations by Burkert (2000) and Kochanek & White (2000) obtained with a constant cross section.

• In the hierarchical framework, the halo core catastrophe is avoided from the balance between mass aggregation process induced by gravitation and thermalization process induced by collisions. If the balance fails, collisions between particles induce core collapse in a short time. Large cross section values cause even faster core collapse.

• If the evidence of soft cores is accepted for Cl 0024+1654 and Abell 1795, the observed central densities of both clusters and dwarf galaxies are reproduced assuming a cross section value of: $(\sigma/m_x)\cdot v_{100}\approx 10^{-24}$ cm^2 GeV^{-1}. The shape and the value of the cross section suggested by our work are in the range of values in which the core collapse and the evaporation problem are avoided (Gnedin & Ostriker 2001; Hennawi & Ostriker 2002).

Observations of the mass distribution at the centre of galaxy clusters are crucial for the debate regarding the nature of the dark matter. If future observations cannot confirm the evidence for soft cores on galaxy cluster scales, then the soft core question remains confined to dwarf and LSB galaxies. In this case different mechanisms could play a role in producing soft cores in dark matter dominated galaxies.

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